End Semester Examination
Electrodynamics

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Duration: 180 minutes
Total points: 90.

Please give arguments where necessary. If it is unclear from your answer why a particular step is being taken, full credit will not be awarded. Grades will be awarded not only based on what final answer you get, but also on the intermediate steps.

1. (a) Find the magnetic field at a distance z above the center of a square loop of side $2a$ carrying current I (take the loop to be lying in the $x-y$ plane, without loss of generality).

   (b) Find the vector potential at a distance r from a long straight wire of radius $a \ll r$ carrying a uniform steady current I. You may use any gauge you choose.

   7 + 8 = 15 points.

2. (a) An electron, with charge $-|e|$ and mass m, executes cyclotron motion (i.e., circular motion) in the $x-y$ plane in a uniform magnetic field $\vec{B} = B\hat{e}_z$. The magnetic field steadily increases at a rate $\frac{dB}{dt}$. At t = 0, the electron has a velocity $\vec{v} = v\hat{e}_x$. Find its tangential acceleration.

   (b) A time-independent surface current $\vec{K} = -K\hat{e}_r$ flows in radially symmetrically from infinity along the $x-y$ plane to the point $r = 0$. The charge accumulates at the origin at the rate $\frac{dq}{dt} = I$.

      i. Find the displacement current.

      ii. Find the magnetic field everywhere.

   6 + (4 + 5) = 15 points

3. The hollow space between two concentric spherical surfaces, of radii a and b $(a < b)$ is filled with a dielectric with a position-dependent $\varepsilon(\vec{r}) = \frac{\varepsilon_0}{1+Kr}$, r being the radial distance measured from the common center. The inner surface carries a total charge Q while the outer surface is grounded. Find

(a) The displacement field in the region $a < r < b$.

(b) The capacitance of the device.

(c) The polarization charge density in the region $a < r < b$.

(d) The surface polarization charge density at $r = a$ and $r = b$.

3 + 6 + 2 + (2 + 2) = 15 points

4. A sphere of radius a and dielectric constant ε_1 is placed in a liquid of dielectric constant ε_2. The liquid is present through all space. A uniform electric field $\vec{E}_0$ is present everywhere. The potential inside can be written as $\varphi(r < a) = \sum_{n=0}^{\infty} \left(A_n r^n + B_n r^{-(n+1)}\right) P_n(\cos\theta)$, while the potential outside can be written as $\varphi(r > a) = \sum_{n=0}^{\infty} \left(C_n r^n + D_n r^{-(n+1)}\right) P_n(\cos\theta)$, since the problem has azimuthal symmetry about the direction of the electric field. Use proper boundary conditions to find the electric field everywhere. Note: you may also find the electric field by any other method you wish.
15 points

5. Consider a parallel-plate capacitor made of circular conducting plates of radius R. The distance between the plates is d. The voltage difference across the place varies with time as $\varphi = \varphi_0 \cos(\omega t)$. Assume $d \ll R$, so that fringing effects can be ignored. Imagine that the space between the capacitors is filled with a dielectric material of dielectric constant ε.

   (a) Find the magnitude and direction of the electric field in the dielectric-filled region as well as the surface charge densities on the capacitor plates as a function of time (ignore magnetic effects for this part).

   (b) Find the magnitude and direction of the magnetic field in the dielectric region.

   (c) Find the flux of the Poynting vector, i.e., the quantity $\oint_A \vec{S} \cdot d\vec{A}$ over the curved surface A of the dielectric material.

   5 + 5 + 5 = 15 points

6. A cylindrical thin shell has length l and radius a, $a \ll l$. The surface charge density on the surface is given by σ. The shell rotates about its axis with an angular velocity $\omega = kt$, $t \geq 0$. Neglecting fringing effects at the ends of the cylinder, find:

   (a) The total magnetic field inside the cylinder.

   (b) The total electric field inside the cylinder.

   (c) The total electric field energy and the total magnetic field energy inside the cylinder. While calculating the field energies in the dynamic situations, feel free to use the corresponding expressions we learnt about while discussing Poynting's theorem.

   5 + 4 + (3 + 3) = 15 points